

En-route sector buffering capacity

David Dugail, Karl Bilimoria and Eric Feron

24th September 2001

Abstract

Using point mass dynamic model for aircraft, we investigate the efficiency of velocity control in the case of an en-route sector subject to an output rate restriction. Velocity control can only be a temporary solution to the problem of an input rate larger than the output rate. This translates into a maximum number of aircraft that can be controlled before a bottom speed limit is reached. We derive this number, both analytically and through simulations. Two control strategies are simulated and analyzed and their differences are exhibited.

1 Introduction

As Air Traffic Management faces challenges in terms of efficiency of the National Airspace System, we investigate the problem of propagation of delays among sectors. Airport-bound flows may face rate restrictions, due to runway capacity for instance, which tend to spread throughout the system. Figure 1 illustrates such a problem when restrictions for aircraft inbound to a New York airport impact traffic hundreds of miles away in a very short amount of time.

Using simple aircraft and airspace models, we look at different control strategies and simulate them with relevant scenarios. Only speed control is used. Analyses are conducted when properties of interest appear.

2 Goals

The objectives of this study are to better understand the behavior of a one-dimensional sector using only speed control to meter its aircraft. Variables of interest are sector length, speed range, rate restriction and standard deviation of interarrival times.

Our metrics will be mainly the capacity of the sector and the responsiveness to an output rate change. Capacity is defined as follows: it is the number of aircraft which have come in at a rate λ and have come out at the restricted output rate μ_r after speed control. Responsiveness is the time between a change of the output rate restriction μ_r and the change of the actual output rate μ as seen by an observer at the exit of the sector.

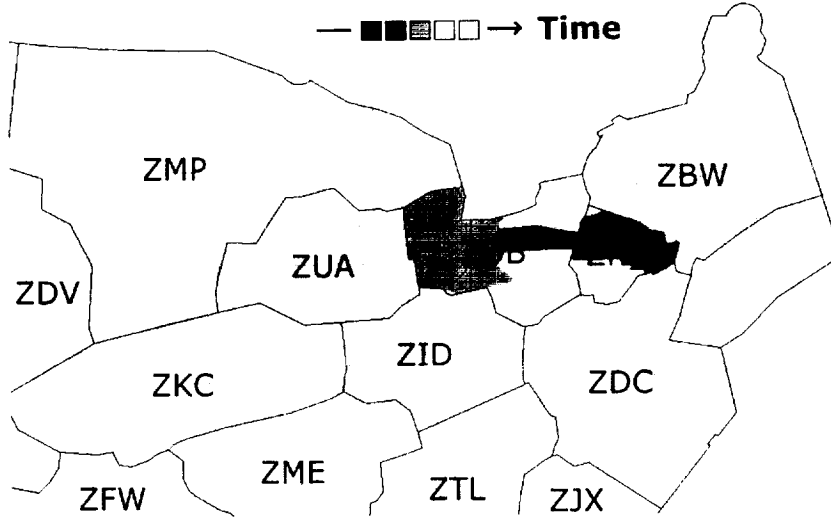


Figure 1: Propagation of a restriction in New York Air Route Traffic Control Center throughout the country. In June 2000, a restriction for Newark-bound traffic propagated further than Chicago ARTCC in less than 20 min.

3 System definition

3.1 Sector geometry

We use a one-dimensional model to represent the en-route sectors. This is justified by the stream-like patterns of airways that are observable close to major airports (Figure 2). However, a lot of sectors also have minor crossing traffic requesting separation: this is not taken into account in our study.

Our sector is thus modeled as an interval of length d , with aircraft coming in at abscissa 0 (entry point “I”) and leaving at abscissa d (exit point “O”). Looking at Figure 2, it is noticeable that most sectors (in the two-dimensional projection) have a dimension a lot larger than the other. This dimension is d , which is typically equal to 150 nm in the National Airspace System. Points I and O represent fixes where flights are handed over from one sector to the next (Figure 3).

3.2 Aircraft

Aircraft are modeled with deterministic kinematics. No dynamics are included and speed changes occur instantaneously. Each aircraft a_i has a state vector position-speed (x_i, v_i) . The aircraft are restricted to fly within a certain speed range, due to buffeting speed limitation on the lower bound and maximum Mach number on the upper bound: $v_i \in [v_{min}, v_{max}]$. A separation zone of $D_{sep} = 5$ nm (FAA standard) is enforced around each aircraft.

Important times in the aircraft journey through the sector are the entry and exit times, denoted t_i and s_i , respectively.

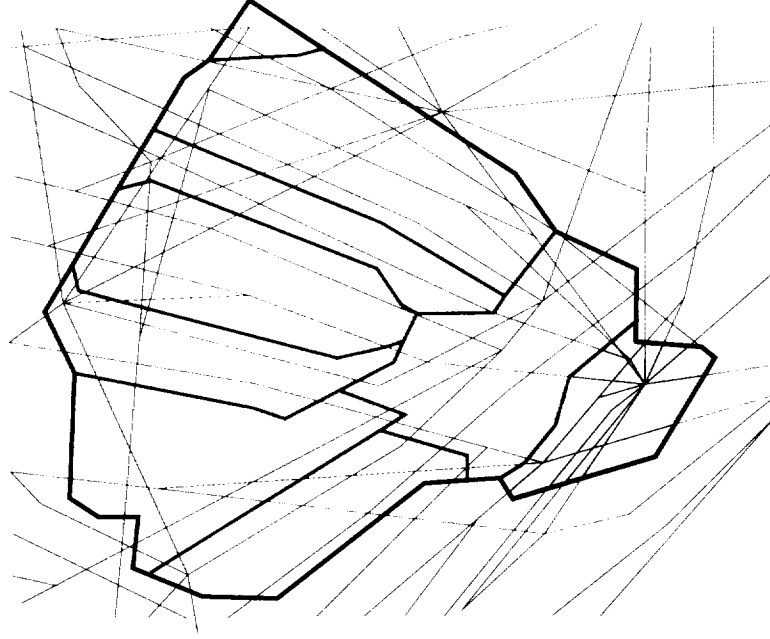


Figure 2: Layout of the New York Center Sector (ZNY). All three major New York airports are located in the yellow area.

3.3 Flows

Aircraft flow comes into the sector at point I (the aircraft are handed over from the upstream sector) and exits at point O (the aircraft are handed over to the downstream sector). The entry rate is λ and the output rate is μ . Input rate λ corresponds, in the real world, to the output rate μ of an upstream sector. If we index the sectors with respect to their streamwise position, we thus have $\lambda^{k+1} = \mu^k$ for all k .

Considering only one sector, μ is dictated from outside, while λ can be controlled. This models the upstream propagation of rate restriction throughout sectors. Because a restriction on μ cannot usually be respected instantaneously, we denote μ_r the desired output rate (desired by the downstream sector). μ is the achieved output rate which should ideally equal μ_r .

Given an input flow λ , aircraft arrival times are modeled in two ways. First, in a deterministic fashion, aircraft interarrival times τ are constant when λ is constant and equal to $1/\lambda = \tau$. To explore the robustness of the results we get, we also inquired about interarrival times normally distributed according to $N[1/\lambda, \sigma^2]$ when σ is the standard time deviation of the distribution.

It should be noted that it is impossible to keep an input rate λ higher than μ_r for an extended time. Drawing a parallel with the principle of mass conservation, we have “aircraft conservation” in the sector, meaning that, in steady state, there must be as many aircraft going out as aircraft coming in (we assume this sector is an en-route sector, thus has no airports to act as “sinks” of aircraft).

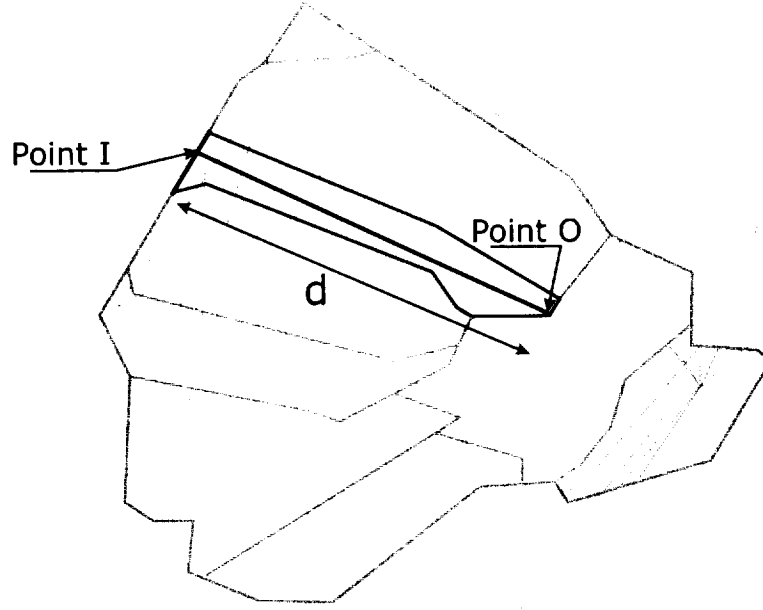


Figure 3: Layout of the New York Center Sector (ZNY). One sector has been singled out to show how our model mimics the sectors with some realism.

4 Control law

Two variations of the law of control are being investigated in our study. As both proceed from the same principle, we describe extensively the first one. The second one is a simple and straightforward modification of the former.

4.1 Entry control

The control system that is implemented in the “entry control” law commands incoming aircraft at the entry of the sector. Aircraft are given a speed command only once. This command cannot be modified later while the aircraft is in the middle of the sector.

The speed command given by the controller is computed out of the enforced output rate restriction μ_r , and former aircraft's (a_{i-1}) speed and position. The speed is adjusted (Equation 1) so that maximum efficiency is achieved, meaning that all aircraft pairs are exactly a time $1/\mu_r$ apart at the output.

However, separation must also be maintained throughout the sector: this requirement can be verified by only enforcing the separation at the exit. Since only one speed command is given at the entry, the separation between any two aircraft in the sector varies linearly with time. If the separation is guaranteed at the exit (by an accurate speed command) and at the entry (assuming the aircraft enter the sector separated), then by continuity the separation is verified throughout the sector. (Equation 2)

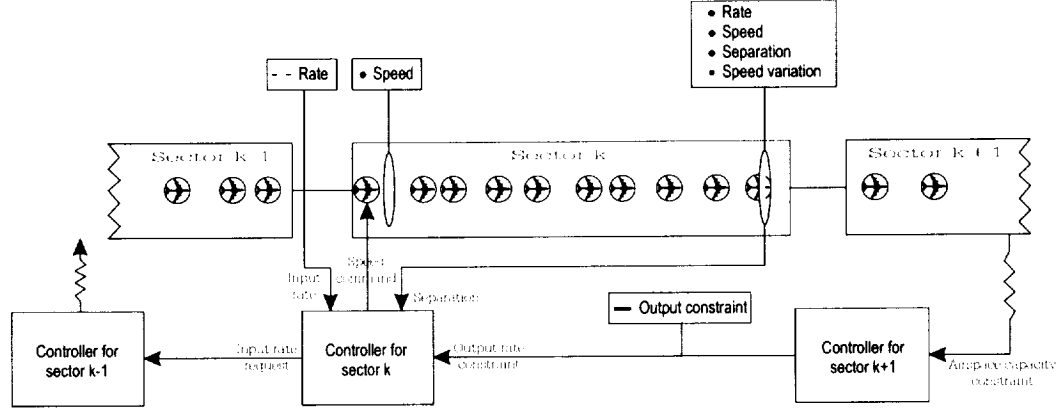


Figure 4: Synopsis of the the system. Measurement points are represented with color markers consistent with those shown on plots.

Of both constraints above, the most important is the separation. Thus, the speed to be commanded to the aircraft must never be higher than that computed in Eq. 2. (Equation 3)

Finally, speed must remain within the operational range of the aircraft. (Equation 4)

All these requirements translate into the following expressions:

$$v^r = \frac{d}{\frac{d-x_{i-1}}{v_{i-1}} + \frac{1}{\mu_r}} \quad (1)$$

$$v^s = v_{i-1} \frac{d}{d + D_{sep} - x_{i-1}} \quad (2)$$

$$v^m = \min(v^r, v^s) \quad (3)$$

$$v_i = \text{sat}_{v_{min}}^{v_{max}}(v^m) \quad (4)$$

where the saturation function is defined as:

$$\text{sat}_a^b(x) = a \text{ for } x \leq a \quad (5)$$

$$= x \text{ for } a < x < b \quad (6)$$

$$= b \text{ for } x \geq b \quad (7)$$

Thus, v_i is the speed command effectively passed to the aircraft. A synopsis of the system appears on Figure 4, where points of measurement collected in the simulations below are also shown.

4.2 Extended control

In this second strategy of control, aircraft are now submitted to the controller's command while they are flying over the first α % of the sector. This means that a change of speed can happen at any time at the beginning of the flight over the sector. However, as long as the output rate does not change, the controls are similar to those given by "entry control." The same requirements as before are in place, leading to the following system of equations:

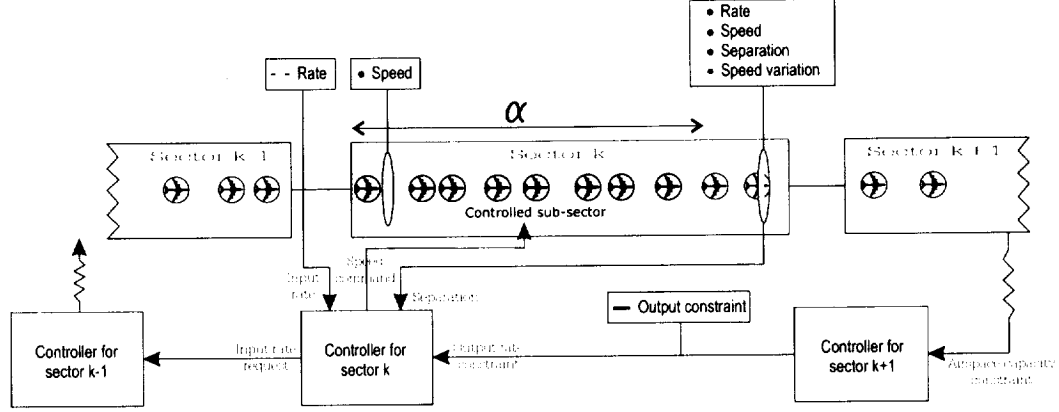


Figure 5: Setup of the extended control law, which includes the possibility to control a portion α of the sector.

$$v^r = \frac{d - x_i}{\frac{d - x_{i-1}}{v_{i-1}} + \frac{1}{\mu_r}} \quad (8)$$

$$v^s = v_{i-1} \frac{d - x_i}{d + D_{sep} - x_{i-1}} \quad (9)$$

$$v^m = \min(v^r, v^s) \quad (10)$$

$$v_i = \text{sat}_{v_{min}}^{v_{max}}(v^m) \quad (11)$$

If $x_i \in [0, \alpha d]$ then v_i is the speed command given to aircraft a_i .

As the information relative to the output rate restriction is influencing the speed of aircraft further into their journey in the sector, the responsiveness of the sector to a change in μ_r can be expected to be better than for “entry control.”

An overview schematic of the extended law is given on Figure 5.

5 Input control

When the sector is not able to deal with an output restriction, it may either decide to violate the output restriction, or request a lower input rate from its upstream sector. For obvious reasons, we are more interested in the latter option.

However, various ways to request a lower input rate are possible. “ λ ” could be requested to be well below, exactly equal to or slightly above (but then for a limited time) the requested output rate μ_r . We have chosen to decrease this input rate to the output restriction at the time when minimum speed has been reached.

This strategy back-propagates the restriction as late in time as possible, though the drawback is a relatively higher restriction than, say, if the restriction had been back-propagated right from the

start. However, the latter strategy would only delay more the back-propagation of the restriction as only $\lambda \leq \mu_r$ is bearable in steady-state.

6 Simulations

Simulations of the system have been written using the Matlab programming environment. In all the examples shown below, the sector of interest is 150 nm long, consistent with today's National Airspace System's design. Airspeed available to aircraft ranges from 400 kt (buffeting speed at cruise altitude) to 500 kt (maximum speed).

A number of parameters are monitored as functions of time. Figure 4 shows a synoptic view of the system including the points where measurements are made. The following subsection details the plotted values.

6.1 Reading the graphs

The simulation results are shown with 3 different plots (see Figure 6 below for illustration). The top plot shows the speed given to the aircraft at the entrance of the sector (with blue dots, one dot representing one aircraft) and the speed at the exit of the sector (with purple dots). One data is not a simple translation in time of the other because an aircraft flying at 400 kt will take more time to pass through the sector than an aircraft flying at 500 kt. A way to read this plot is to see the two lines as representing the range of speeds of the aircraft within the sector. For instance, at $t=2$ hr on Figure 6, aircraft in the sector have speeds ranging from 415 kt to 465 kt approximately.

The middle plot shows a number of different rates of aircraft (i.e. the number of aircraft going through certain points over an hour): the dotted green line represents the input rate λ from the upstream sector (sector $k-1$ as labeled in Figure 4), the solid red line the restriction μ_r on the output flow as requested by the downstream sector (sector $k+1$). The blue dots are the actual output rate μ . These turn red if the rate is in violation of the output constraint. However, because in this model we "flush" the first aircraft, some blue dots appear as being above the red line. These represent the "flushed" aircraft.

The bottom plot shows two different things: first, with blue dots, the separation of aircraft at the exit is shown. This separation, in nautical miles (nm) should never be below 5 nm for safety reasons. If the separation is greater or equal to 5 nm at the exit, it is assured that the separation between aircraft is also above the safety minimum during the whole time within the sector (see Section 4.1). Finally, a graph consisting of green dots shows the difference in speed of two consecutive aircraft.

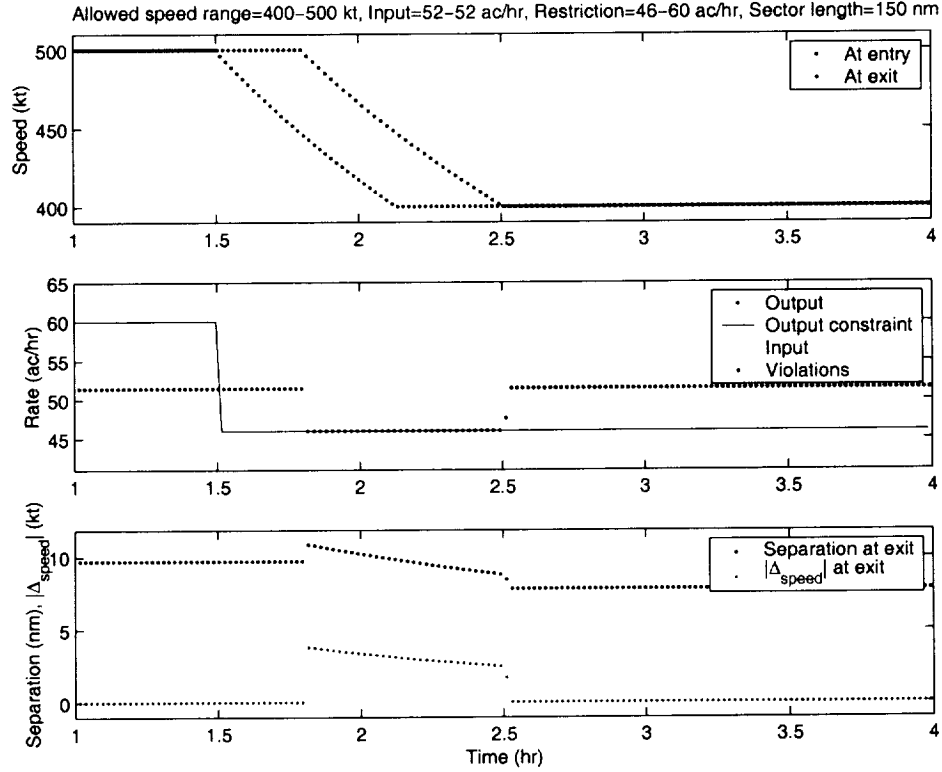


Figure 6: Simulation with deterministic scheduled arrivals. A restriction is imposed on the output at $t=1.5$ hr.

6.2 Saturation of the sector's capacity

6.2.1 Deterministic with entry control

In this first simulation (Figure 6), we test the endurance of the system when an output restriction lower than the input rate lasts for a “long” time, long enough to witness a saturation of the system.

The situation is initially at steady-state with 52 arriving aircraft per hour ($\lambda = 52$), at a speed of $v=500$ kt, thus creating a spacing of $D = v/\lambda = 9.6$ nm. The output restriction is set to $\mu_r = 60$, thus above the input rate λ or, equivalently, no restriction is imposed. There is no speed command given to the incoming aircraft.

At $t=1.5$ hr, the output restriction goes below the input rate at $\mu_r = 46$, thus triggering the need for speed control of incoming aircraft. Entering aircraft are slowed down and any new aircraft goes slower than those preceding. This control only impacts the output rate after the time needed for the first impacted aircraft to reach the end of the sector, approximately $t = d/v = 18$ min. From there on, until the speed command reaches an unacceptably low level ($v_{min} = 400$ kt), the output rate matches the output rate constraint. When this level is reached, the new incoming aircraft are only given the command to slow down to v_{min} , in order to maintain spacing, but the output μ rate will come back up to λ (all new aircraft are given the exact same speed when entering, thus their output time spacing remains their input time spacing), thus violating the output restriction.

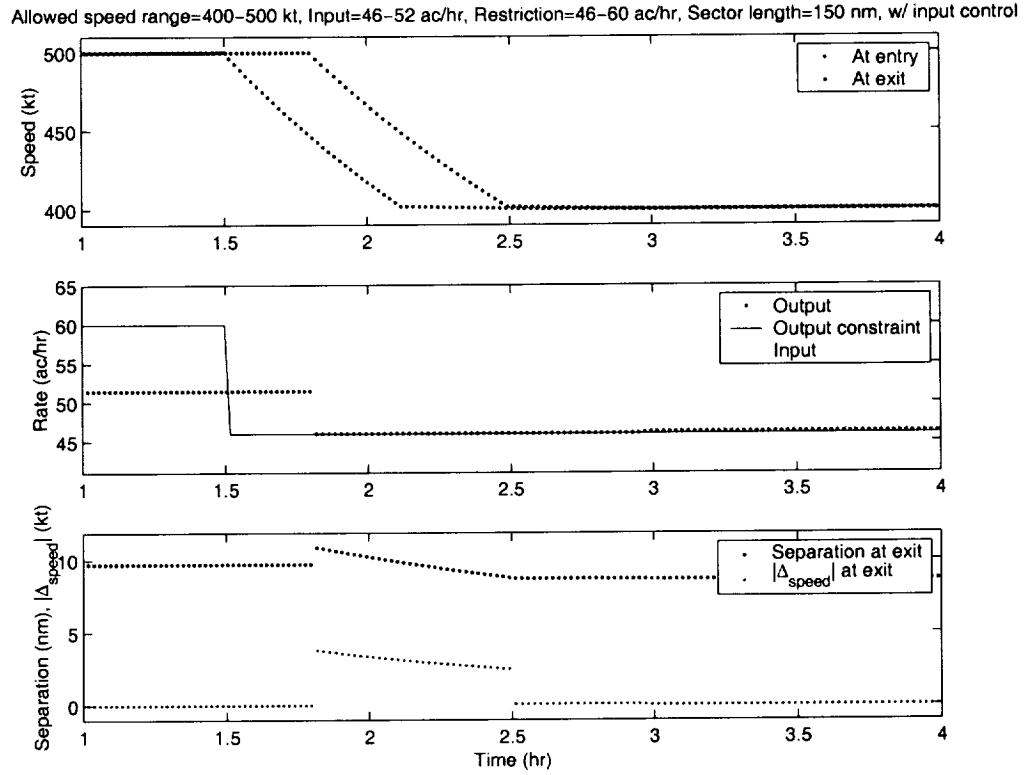


Figure 7: Simulation with deterministic scheduled arrivals. A restriction is imposed on the output at $t=1.5$ hr. Input control is active and lowers the input rate at $t \approx 2.2$ hr.

6.2.2 Deterministic with entry and input rate control

One way to avoid this violation is to back-propagate the restriction to the upstream sector, and impose a lower input rate at some point in time (see Figure 7): this mimics what is done in the real ATM world. Here is implemented the control described in Section 5.

Because the input rate exactly matches the output rate, speed control is unnecessary and impossible, thus prohibiting the speed of the flow to build up again. To see the speed command increase, one needs some extra time spacing, provided either by a lower input rate or a higher output rate.

6.2.3 Randomized arrivals with entry control

In this section, we present a simulation analogous to that of Figure 6, except this time the arrival process is a randomized one (Figure 8). The random variable here is the time between two successive arrivals. This inter-arrival time is normally distributed with a mean equal to $1/\lambda$, λ being shown on the plot by the dotted green line, and a standard deviation $\sigma = 5$ s. We then know that 96% of all inter-arrival times are going to occur in $[1/\lambda - 3\sigma; 1/\lambda + 3\sigma]$.

It is interesting to notice that the capacity, defined as the number of aircraft passing through the sector and able to match the desired output constraint, is not much impacted by the randomness

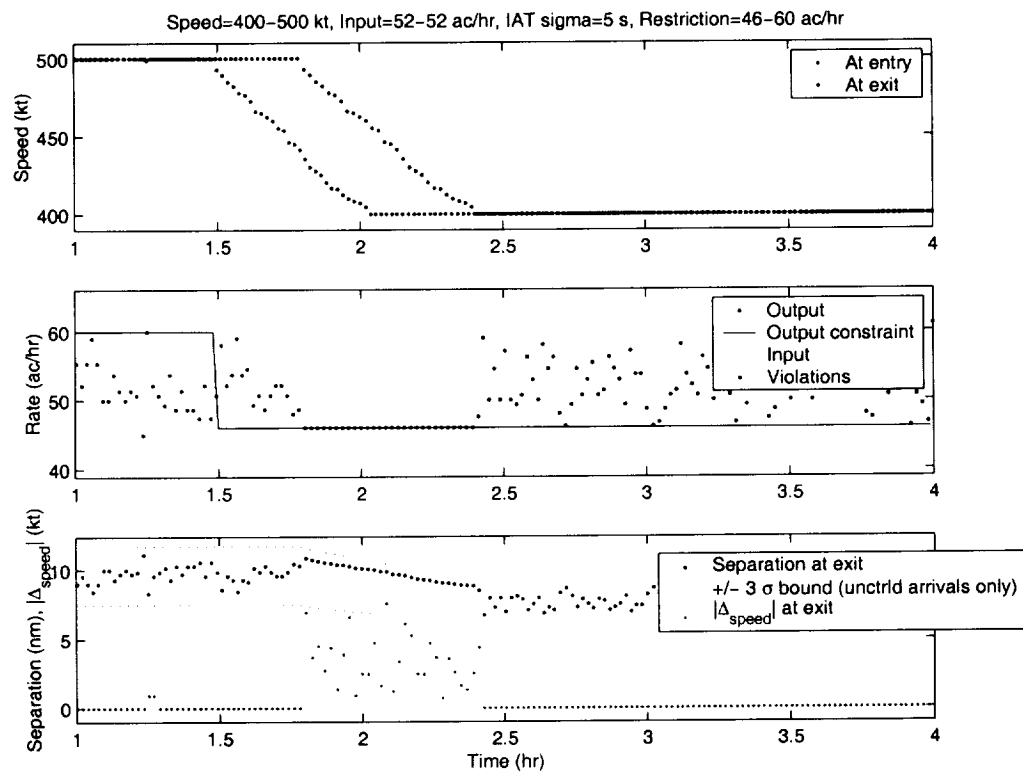


Figure 8: Simulation with randomized arrivals. A restriction is imposed on the output at $t=1.5$ hr.

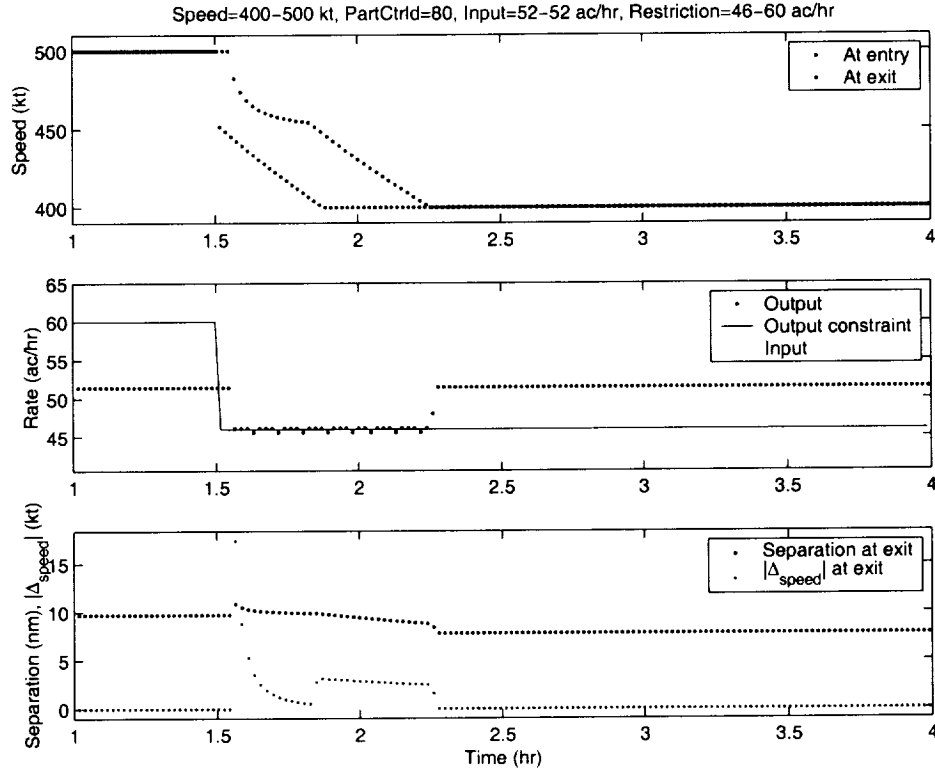


Figure 9: Simulation with 80% of the sector controlled. A restriction is imposed on the output at $t = 1.5$ hr.

of the arrivals. Multiple runs of simulations with randomized arrivals have shown that the average capacity remains constant for different values of sigma, as long as the randomness does not imply a violation of the minimum separation constraint at the entry.

6.2.4 Deterministic with extended control

An output restriction might often be imposed by the downstream sector with short notice. The faster the sector can respond to this output constraint, the better. In order to improve the response of the system ($t = d/v$), we are now considering a sector controlling the first $\alpha\%$ of the sector (Figure 5). We may not control 100% of the sector because an aircraft just about to leave the sector would have to make a dramatic speed decrease over a very short time to cope with a relatively small delay. Not only is the result not worth the pain, it is also barely sensible to model the aircraft speed change as instantaneous.

We first show a simulation using this particular algorithm (see Figure 9) where $\alpha = 80\%$ of the sector is now controlled. The time of response to the restriction is now much lower. Straight-forward analysis tells us that the 20% of the sector not under control translates into a delay of $t = (1 - \alpha)d/v$ (or approximately 4 min) before the restriction is met.

Also noticeable are the speed curves. The speed at exit from $t \simeq 1.6$ hr to $t \simeq 1.8$ hr has a particular

shape, different from that seen before. It reflects the speed commands given to the aircraft already in the sector when the restriction was issued. Once those aircraft have been flushed, the usual variation returns. From the entry speed standpoint, it is obvious that all the commands given to aircraft inside the sector are ignored. The only information passed back is the speed of the aircraft closest to the entry at the time of the restriction.

A very interesting property appears when modifying the parameter α . For all α 's, as long as the speed commands given to aircraft within the sector at the issuance of the restriction lie in the acceptable range of speed (i.e., greater than v_{min}), the capacity of the sector remains unchanged. This means that controlling a greater portion of the sector does not, in this particular case, induce something else than faster response from a Traffic Management perspective. An analysis of that property is given below (Section 7.2).

6.3 Temporary output restriction

After a restriction has had to be met for some time, and this restriction has been lifted to a value higher than the input rate, speed commands should be driven back to their original value (here $v=500$ kt). However, two parameters influence this process, hereafter called recovery: the minimum separation distance between aircraft which must be verified at all times, and the new output rate which still must be met (see Section 4.1).

In the next study, either the output restriction is sufficiently close to the input rate, or the duration of this restriction is sufficiently short to avoid the minimum speed limit to be reached. Both modes of recovery explained above are identified.

First, on Figure 10, this recovery is driven by the output rate: the speed command is high enough that aircraft are time spaced at the output at the lowest value admissible for the given output rate (here: $\mu_r=60$). Separation remains above the 5 nm limit, and thus does not constitute a hard bound.

On the contrary, on Figure 11, speed commands are given so that the 5 nm spacing is respected throughout the sector. If speed commands were given accounting only on the possible output rate, this separation would have been violated. Thus, the actual output rate is lower than the maximum allowed.

7 Analyses

Two main points have been emphasized for analysis purposes. In the following sections, we dwell on the following analyses: computation of the capacity and difference between entry control and extended control situations.

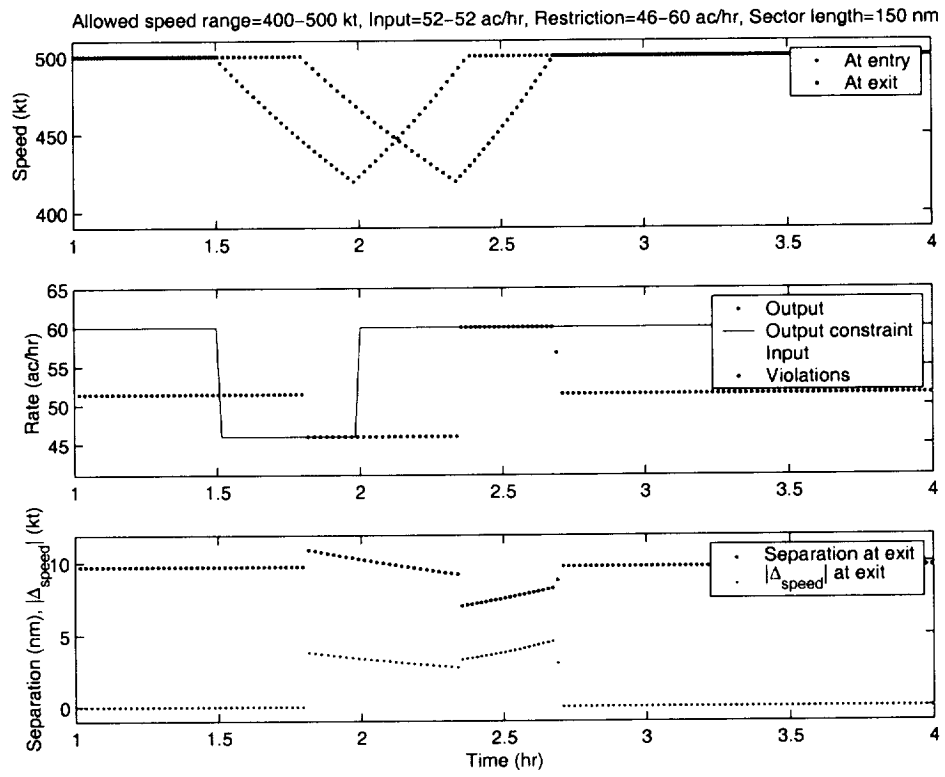


Figure 10: Simulation with deterministic scheduled arrivals. A restriction is imposed on the output at $t=1.5$ hr and lifted at $t=2$ hr.

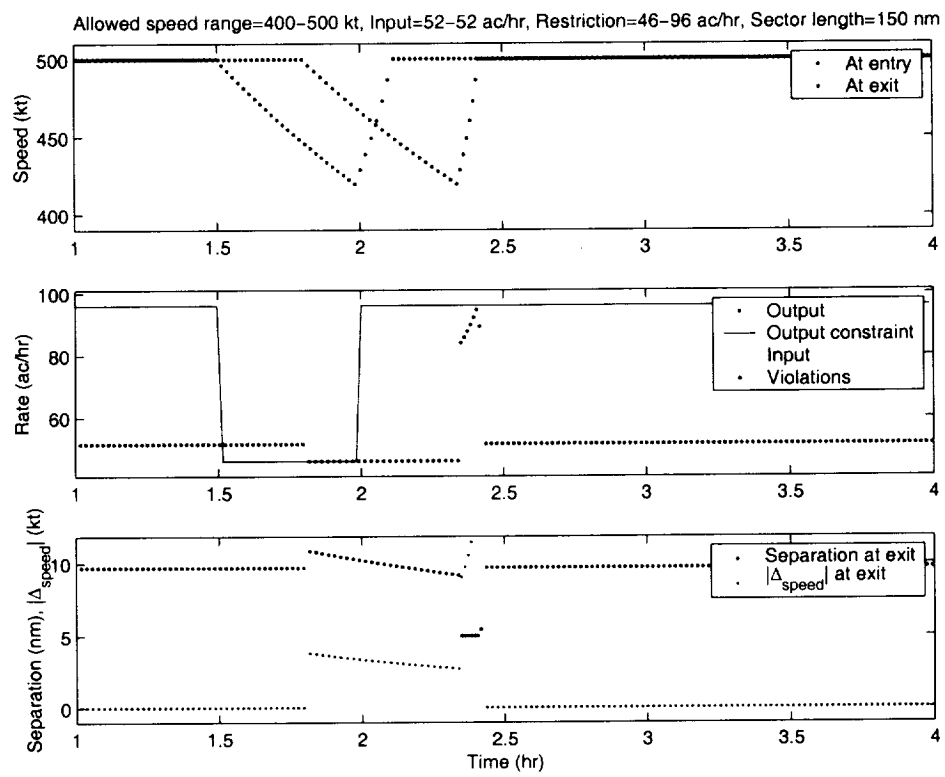


Figure 11: Simulation with deterministic scheduled arrivals. A restriction is imposed on the output at $t=1.5$ hr and lifted at $t=2$ hr.

7.1 Capacity with entry control

Capacity computation can be achieved in two different ways. One is to use Equation 1 recursively, in order to compute the speed of the n^{th} aircraft when a restriction μ_r is enforced, with a flow initially flying at v_f . Once v_n is known, capacity is computed by solving $v_n = v_{\min}$ for n . However, closed-form results will be hard to give, due to the recursive nature of the calculation.

Another way to proceed is to see aircraft as items with scheduled times of entry and exit. Knowing those two times, speed is easily derived knowing the length of the sector.

Let's constrain our analysis to entry control only under deterministic arrivals; we will see later that the following is also true for the extended control case. Using the definitions for t_i (time of entry) and s_i (time of exit) given in Section 3.2, we come up with the following.

The metering constraint μ_r is set at time T . Thus, for all aircraft a_i such that $t_i > T$, the exit time s_i is set so that $s_i - s_{i-1} = 1/\mu_r$. Let's assume $T = t_0^+$ or, said differently, the constraint is set at the time of entry of aircraft a_0 , but the speed command given to a_0 has not taken into account the restriction μ_r . Thus:

$$\begin{aligned} s_0 &= T + \frac{d}{v_0} \\ &= S \\ s_1 &= T + \frac{d}{v_1} \\ &= \frac{1}{\mu_r} + S \\ s_n &= \frac{n}{\mu_r} + S \end{aligned} \tag{12}$$

On the other hand:

$$t_n = \frac{n}{\lambda} + T \tag{13}$$

Combining Equations 12 and 13, we come up with the expression for v_n :

$$v_n = \frac{d}{n \left(\frac{1}{\mu_r} - \frac{1}{\lambda} \right) + \frac{d}{v_f}} \tag{14}$$

Equating (14) with v_{\min} , we can solve for n , the capacity of the sector:

$$n = \frac{d \left(1 - \frac{v_{\min}}{v_f} \right)}{v_{\min} \left(\frac{1}{\mu_r} - \frac{1}{\lambda} \right)} \tag{15}$$

As expected, the capacity of the sector depends on the sector length d , the flow speed v_f and the minimum speed v_{\min} and the input and output restriction rates.

7.2 Capacity with extended control

We saw earlier in the simulation implying extended control (Section 6.2.4) that capacity did not change with the portion of sector controlled by the algorithm.

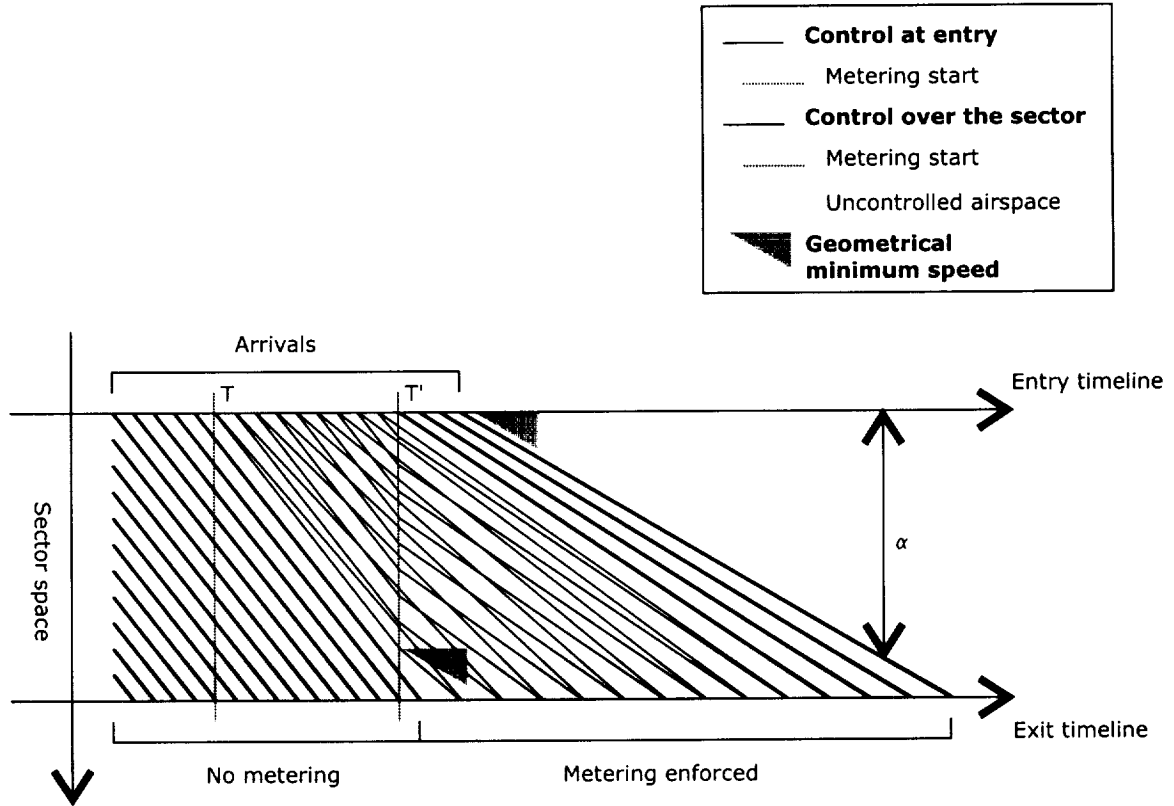


Figure 12: Geometrical approach to understand why capacity does not change with α , the controlled proportion of a sector. Shown in black and red solid lines are the time-space trajectories of aircraft either under single entry control or control over α of the sector.

A visual proof of that property appears on Figure 12. Let's first have a look at the trajectories in red. These are the trajectories created by aircraft controlled under entry control only. The restriction is issued at T. From there on, entering aircraft are assigned times of exit, separated in accordance with μ_r . Accordingly, speed commands that are expected to respect these schedules are given. This lasts as long as the minimum speed, materialized by a green triangle on the picture, is not reached.

If we now turn to black trajectories (control over α % of the sector), what we see is that, mainly, the trajectories and scheduled times of exit are identical. Only the time of issuance of the restriction T' appears later than for the previous case, which means it is closer to the time when the metering is enforced (already noticed with the faster response). Trajectories are identical except for those which intercept the line representing time T' (i.e., the aircraft already in the sector at time T'). These trajectories have "kinks" in them, resulting from the change of speed given to them at this time. However, they have the same scheduled times (with another time reference, though), and as such do not interfere differently with the trajectories of the aircraft still out of the sector at T'.

The evolution after the first aircraft have been flushed away from the sector is identical in both cases and maximum capacity is reached at the same time, when v_{min} (represented by the triangle) is reached. Thus both control laws yield the same capacity.

We recall Equation 15 of capacity, which still applies for the extended control case:

$$n = \frac{d \left(1 - \frac{v_{min}}{v_f}\right)}{v_{min} \left(\frac{1}{\mu_r} - \frac{1}{\lambda}\right)}$$

8 Conclusion

This report provides a few examples of how velocity control can help delay the propagation of restrictions through the National Airspace System. In some cases, a relief of only a few minutes can help avoid cascading effects happening, such as the situation shown in the introduction. Under some idealizing assumptions, we have been able to derive analytically the delay (in number of aircraft) provided by velocity control. This result relies on a deterministic model of aircraft dynamics and could be improved greatly (in the sense of getting closer to reality) by taking a more stochastic approach. However, we hope our results will help build some understanding about the way velocity control can prevent dramatic congestion of the airspace.

PROF. ERIC FERON
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
77 MASSACHUSETTS AVE.
35-417
CAMBRIDGE, MA 02139



9/24/01

Dr. Karl Bilimoria, Technical Monitor
NASA-Ames Research Center
Automation Concepts Research Br., 210-10
Moffett Field, CA 94035

Ref: NASA Cooperative Agreement #NCC2-1209 (OSP 6891597)

Dr. Bilimoria,

Please find enclosed the annual progress report for the above-referenced cooperative agreement covering the period November 15, 2000 to November 14, 2001.

Best Regards,

Eric Feron

Cc:
w/encl:
Document Processing Section, NASA

Cc:
wo/encl:
Venoncia Braxton, NASA-Ames Research Center
Scott E. Nirk, Office of Naval Research
Lauren Gallant, MIT
Ping Lee, MIT
Charlotte Morse, MIT